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ON THE JOINT DYNAMICS OF POLLUTION AND CAPITAL ACCUMULATION

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Abstract
I construct an overlapping generations model in which longevity is impeded by the stock of pollution and promoted by public health spending. I provide an alternative explanation for the so-called environmental Kuznets curve – an explanation which gives an active role to environmental quality as a contributing factor to capital accumulation and growth. I also examine how variations in environment-related parameters determine the effect of taxation in economic development.

JEL classification: O41; Q56

Keywords: Overlapping generations; Pollution; Capital accumulation; Endogenous longevity

1 Introduction

As the interest on the underlying characteristics and driving forces of economic growth rose considerably during the last two decades, it was inevitable that many researchers would turn their attention to the implications of sustained growth for the quality of the natural environment. The fact that various by-products of economic activity (e.g., chemicals, toxins,

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smoke, radioactive substances and litter) contaminate and erode the natural environment cannot be disputed. Yet, some authors have questioned the view that sustained economic growth may ultimately prove to be the source of natural catastrophe caused by unbounded environmental degradation. Their arguments are based on statistical evidence which, in the words of Brock and Scott-Taylor (2004), shows the "tendency for the environment to at first worsen at low levels of income but then improve at higher incomes" (Brock and Scott-Taylor, 2004; p. 3).[1]

Naturally, the identification of the determinants behind an inverse U-shaped relationship between various measures of pollution and per capita GDP took a prominent place in theoretical analyses that incorporated elements of environmental quality in otherwise standard economic frameworks. In a static model, Andreoni and Levinson (2001) attribute the emergence of the environmental Kuznets curve to the presence of increasing returns in the technology of pollution abatement: since output contributes to both environmental degradation (through the emission of pollutants) and pollution abatement, the presence of increasing returns in the latter's technology can reproduce the functional relationship that is effectively the environmental Kuznets curve. In the dynamic frameworks of Stokey (1998) and Hartman and Kwon (2005), a central planner finds optimal to initiate pollution control, through the introduction of 'cleaner' technologies, only after a threshold level of output is surpassed. From that point onwards, it is possible for pollution to decline constantly at a rate which is proportional to the rate of output growth.[2] A similar story emerges in the overlapping generations framework of John and Peccherino (1994): although the initial stages of growth are associated with environmental deterioration due to the absence of pollution abatement, once the latter is implemented the environment may improve because economic growth supports abatement techniques.[3]

[1] For evidence on air pollutants such as sulphur dioxides, nitrogen oxides and smoke, see Millimet *et al.* (2003) and Aslanidis and Xepapadeas (2008) among others. Grossman and Krueger (1995) provide support for a number of water pollutants as well (e.g., chemical oxygen demand, biochemical oxygen demand and some heavy metals such as arsenic).

[2] The models by Stokey (1998) and Hartman and Kwon (2005) differ in their implications concerning the sustainability of the endogenously derived balanced growth path.

[3] See Kelly (2003) for a model that examines how variations in structural parameters may affect the shape of the relationship between emissions and capital accumulation (including the possibility of an environmental Kuznets curve).

A common feature to all these analyses is the explanatory power of output movements in single-handedly determining the variations of pollution and environmental quality. Thus, given that the response of pollution to increasing levels of income eventually changes sign, the environmental Kuznets curve seems to imply that, as output grows and reaches sufficiently high levels, pollution will virtually disappear. Yet, under what circumstances could these implications emerge? The first one is the idea of modelling the environmental impacts of economic activity and pollution abatement as being separable. Normally, however, abatement is associated with activities that serve in mitigating pollution, so the suggestion that such activities may be able to override its negative impact altogether seems implausible. An even more crucial assumption has to do with the economy's apparently unlimited scope in reducing its emission rate. Although many developed economies have been successful in reducing their emissions per unit of produced output (e.g., shifts in the composition of production, outsourcing, environmental regulation, environment-related R&D etc.) the fact still remains that, in all probability, the elimination of all the polluting by-products of economic activity is nothing more than wishful thinking – at least for the foreseeable future. This is an important point because, insofar as the 'cleanest' possible technology still emits some pollutants, no matter how low these are, the implication would be that the growth process (by itself) will not be able to justify continuous reductions in the level of pollution – an idea that is important for the existence of a U-type relationship between environmental quality and GDP.

This paper aims at providing an alternative explanation for the observed pattern of co-movements between output and various measures of pollution. Taking account of the preceding arguments, it focuses on steady-state *levels* rather than steady-state (i.e., sustained) *growth* for per capita income. Nevertheless, it is still able to reproduce co-movements for environmental quality and income per worker that constitute an 'environmental Kuznets curve', as it identifies them in terms of the dynamics experienced by an economy when it transits between two steady-state regimes for the stocks of physical capital and pollution. Ultimately, the present framework contributes to the ongoing debate by proposing the idea that environmental quality is a (partially) contributing rather than a passive factor in the possible emergence of an environmental Kuznets curve – an idea that has, surprisingly, eluded the attention of researchers.

The novelty of the approach lies on the explicit account of the by-directional effects between economic activity and environmental quality. Specifically, I consider the positive repercussions of reduced pollution for economic outcomes in addition to production's contribution to environmental degradation. I do this by utilising the idea that an improved environment contributes to a rise in life expectancy which, subsequently, promotes saving behaviour and capital accumulation. Moreover, I utilise the idea of threshold effects in the process of economic development – the threshold being identified as the level of income at which a 'cleaner' production method is implemented. The environmental Kuznets curve is then explained in terms of the transitional dynamics of an economy which, following a permanent structural change, moves from an original equilibrium below the threshold to a new equilibrium which is situated above this threshold. During the initial stages of this transition, capital accumulation leads to output growth which, for a given emission rate, causes pollution to rise. When the threshold is reached, however, the emission rate falls and leads to a new dynamic adjustment: from that point onwards it is the reduction in pollution that is largely responsible for output's further growth towards the new equilibrium, as the improved environment reduces the risk of premature death and (by increasing the saving rate) promotes the accumulation of capital.

Following the tradition set by Barro (1990), among others, the paper also aims at providing conditions for which taxation can be conducive to economic development, as long as its proceeds are committed to productive purposes. In this framework, the productive use of tax receipts takes the form of publicly provided services towards health care – a policy that raises life expectancy. I show that, under a reasonable specification for the probability of survival, the tax rate that maximises the economy's long-term development prospects is increasing in the parameters that cause and/or exacerbate environmental deterioration.

The remaining paper is structured as follows: in Section 2 I present the fundamental characteristics of the economy. In Section 3 I derive the steady-state equilibrium, check its stability and undertake the analysis of some comparative statics. Section 4 considers the implementation of less polluting production methods and shows how the environmental Kuznets curve can be attributed to the joint transitional dynamics and by-directional effects of pollution and capital accumulation, following a permanent structural change that promotes the formation of capital. In Section 5 I analyse the rate of public health spending which is most favourable to the prospects of economic development. Section 6 concludes.

2 The Economy

Time takes the form of discrete periods which are indexed by t and measured from zero to infinity. In each period, there are two cohorts of agents inhabiting the economy – the young and the old. A mass of young agents (whose size I normalise to one) comes into existence at the beginning of each period. Each young agent is endowed with one unit of labour which she supplies inelastically to firms in exchange for the market wage w_t. She then decides how much to consume and how much to save for retirement, given that, when old, she does not have any labour endowment and, therefore, any alternative source of income from which she could finance her future consumption needs.

One deviation of this model from the standard overlapping generations setting (Diamond, 1965) is the idea that survival to maturity is not certain. Instead, survival is determined by the realisation of a mortality shock. Specifically, I assume that a young person will survive to maturity with probability $\psi_t \in (0,1)$, whereas with probability $1-\psi_t$ she dies prematurely and cannot enjoy any activities (mainly, consumption) when old. Provided that only agents who survive are able to consume in both periods, their *ex ante* (i.e., expected) lifetime utility is given by

$$U^t = \ln c_t^t + \psi_t \ln c_{t+1}^t, \tag{1}$$

where c_i^j denotes consumption in period i of an agent born in period j. Each agent maximises her lifetime utility subject to the constraints for consumption during youth and old age. Denoting saving by s_t and the gross rate of interest on deposits by r_{t+1}, these constraints are given by $c_t^t = w_t - s_t$ and $c_{t+1}^t = r_{t+1}s_t$ respectively.

The consumption good is produced and supplied by perfectly competitive firms. These firms hire labour from the young, denoted L_t, and capital from financial intermediaries, denoted K_t, and combine them to produce Y_t units of output according to a neoclassical technology $Y_t = F(K_t, L_t)$ with $F_i > 0$ and $F_{ii} < 0$ for $i = K_t, L_t$.[4] The production function is assumed to be homogeneous of degree one in capital and labour. A functional form that

[4] Capital depreciates completely in production.

satisfies these properties, and that will be employed hereafter, is $Y_t = BK_t^{\beta} L_t^{1-\beta}$ with $B > 0$ and $0 < \beta < 1$. The intensive (i.e., per worker) form is given by

$$y_t = Bk_t^{\beta}, \tag{2}$$

where $y_t = Y_t / L_t$ and $k_t = K_t / L_t$.

All firms are subject to a proportional tax $\tau \in (0,1)$ on their production. Therefore, taking account of (2), profit maximisation yields the familiar conditions

$$R_t = (1-\tau)\beta Bk_t^{\beta-1}, \tag{3}$$

and

$$w_t = (1-\tau)(1-\beta)Bk_t^{\beta}, \tag{4}$$

where R_t is the rental price of capital.

Financial intermediaries undertake the task of channelling capital from households to firms. Specifically, they accept deposits by young agents and, in return, they offer the gross rate r_{t+1}. They transform these saving deposits into capital by accessing a technology that transforms one unit of time-t output into $q > 0$ units of time-$t+1$ capital which they supply to firms at a rental cost of R_{t+1} per unit.[5] Following others (e.g., Chakraborty, 2004; Tang and Zhang, 2007), I appeal to the idea that the young deposit their saving to a mutual fund which promises to provide retirement income, provided that the depositor survives to old age. Otherwise, the income of those who die is shared equally by surviving members of the mutual fund. In view of this, and the assumption that financial intermediaries operate under perfect competition, we have

$$\psi_t r_{t+1} = qR_{t+1}, \tag{5}$$

which implies that their costs (i.e., the total return to all surviving savers) must be equal to their revenues (i.e., the revenues they receive from firms who rent capital).

As already mentioned, the government levies taxes from firms, which amount to revenues of τY_t. The public sector uses these funds to provide goods and services towards public health care. Let us assume that the government abides by a balanced budget rule each period. As a result, we have

[5] We may think of q as the efficiency of the economy (in general) or of the financial sector (in particular) in successfully transforming resources into productive capital.

$$h_t = \tau Y_t . \tag{6}$$

In terms of intuition, public health expenditures may include salaries paid to medical staff (doctors, nurses etc.), maintenance of infrastructure (e.g., hospital buildings), enforcement of laws and regulations that preserve health and safety, and funding of medical research.

2.1 Life Expectancy

I assume that life expectancy, captured by the probability of survival ψ_t, is endogenous.[6] In particular, it takes the form of a function

$$\psi_t = \Psi(x_t), \tag{7}$$

such that $\Psi' > 0$, $\Psi'' < 0$, $\Psi(0) = 0$, $\Psi(\infty) = \lambda \in (0,1)$, $\Psi'(0) = \varphi > 0$ and $\Psi'(\infty) = 0$. I also impose the restriction $\Psi(x_t) > \Psi'(x_t)x_t \quad \forall x_t > 0$.[7] Indicatively, a functional form that satisfies all these properties is $\Psi(x_t) = \frac{\lambda x_t}{1 + x_t}$, with $0 < \lambda < 1$ and $\lambda = \varphi$.

Naturally, the endogeneity of longevity is captured by the term x_t for which I assume that it is related to public spending in health services, denoted h_t, and pollution, denoted μ_t, according to $x_t = X(h_t, \mu_t)$. This satisfies $X_{h_t} > 0$, $X_{\mu_t} < 0$, $X(0, \mu_t) = X(h_t, \infty) = 0$ and $X(\infty, \mu_t) = X(h_t, 0) \to \infty$. I restrict my attention to a specific functional form which is

$$x_t = \frac{h_t}{\mu_t} . \tag{8}$$

Substitution of (8) in (7) yields

$$\psi_t = \Psi\left(\frac{h_t}{\mu_t}\right), \tag{9}$$

such that $\Psi_{h_t} > 0$, $\Psi_{\mu_t} < 0$, $\Psi(h_t = 0) = \Psi(\mu_t \to \infty) = 0$ and $\Psi(h_t \to \infty) = \Psi(\mu_t = 0) = \lambda$. Given $\Psi(x_t) > \Psi'(x_t)x_t$ it also follows that $\Psi_{h_t h_t} < 0$ and $\Psi_{\mu_t \mu_t} > 0$. Once more, all these

[6] The expected life span of a person born in t is $2\psi_t + (1 - \psi_t) = 1 + \psi_t$. For this reason, I will be making use of such terms as 'life expectancy', 'longevity' and 'survival probability' interchangeably.

[7] This assumption, together with a restriction on the relative share of capital that will be imposed later, is essential to guarantee the stability of the economy's long-run equilibrium.

properties are satisfied with the function $\Psi(x_t) = \frac{\lambda x_t}{1+x_t}$ which, after substituting (8), becomes $\Psi(\cdot) = \frac{\lambda h_t}{h_t + \mu_t}$.

In terms of intuition, the basic assumptions concerning life expectancy imply that, *ceteris paribus*, an increase in the provision of health services by the government and/or an improvement in environmental quality (that is, a reduction is the stock of pollutants) should have a beneficial impact on the longevity prospects of the population. These ideas conform to the evidence provided by numerous empirical studies on these issues (e.g., Anand and Ravallion, 1993; Pimentel *et al.*, 1998).

2.2 Pollution

The pollution stock is denoted μ_t and it evolves over time according to

$$\mu_{t+1} = \eta\mu_t + P_t . \tag{10}$$

The parameter $\eta \in (0,1)$ indicates the environment's absorption capacity: higher values of η point to the nature's reduced capacity in mitigating the cumulative impact of the current on the future pollution stock. The variable P_t is the flow of pollution which determines the degrading impact of economic activity on environmental quality. Hence, it is related to total output according to $P_t = pY_t$, where $p > 0$ is an indicator of how 'dirty' the manufacturing process is – i.e., how many pollutant emissions are released into the environment per unit of output produced. Using $P_t = pY_t$ in (10) yields

$$\mu_{t+1} = \eta\mu_t + pY_t . \tag{11}$$

Equation (11) demonstrates how economic activity, combined with the existing level of pollution, contributes further to the decay of the natural environment by adding to the future pollution stock.

3 Equilibrium

Taking account of the fundamental relationships in the economy, we can describe its temporary equilibrium with

Definition 1. *The temporary equilibrium of the economy is a set of quantities* $\{c_t^{t-1}, c_t^t, c_{t+1}^t, s_t, L_t, Y_t, \psi_t, P_t, h_t, \mu_t, K_t, K_{t+1}\}$ *and prices* $\{w_t, \mathrm{R}_t, \mathrm{R}_{t+1}, r_{t+1}\}$ *such that:*

(i) *Given* w_t, ψ_t, r_{t+1} *and* μ_t, *the quantities* c_t^t, c_{t+1}^t *and* s_t *solve the optimisation problem of an agent born at time* t;

(ii) *Given* w_t *and* R_t, *firms choose quantities for* L_t *and* K_t *to maximise profits;*

(iii) *The labour market clears, i.e.,* $L_t = 1$;

(iv) *The goods market clears, i.e.,* $Y_t = c_t^t + \psi_{t-1} c_t^{t-1} + s_t + h_t$;

(v) *The financial market clears, i.e.,* $\psi_t r_{t+1} = q\mathrm{R}_{t+1}$;

(vi) *The government's budget is balanced, i.e.,* $h_t = \tau Y_t$.

The optimisation problem of a young agent leads to a solution for saving given by

$$s_t = \frac{\psi_t}{1+\psi_t} w_t . \tag{12}$$

The possibility of premature death induces the agent to modify her saving behaviour in response to variations in life expectancy. This is because an increase in the probability of survival raises the (expected) marginal utility of consumption when old. To restore the equilibrium, the marginal utility of her consumption when young must increase as well – something that the agent can achieve by saving more and consuming less during the first period of her lifetime.

The equilibrium condition $L_t = 1$ implies that $K_t = k_t$ and $Y_t = y_t$ $\forall t$. Therefore, using (4) and $k_{t+1} = qs_t$, equation (12) becomes

$$k_{t+1} = q\Theta \frac{\psi_t}{1+\psi_t} k_t^{\beta} , \tag{13}$$

where $\Theta = (1-\tau)(1-\beta)B$. Furthermore, we use $Y_t = y_t$ together with (2), (6) and (9), and substitute in (13) to derive the dynamics of capital accumulation according to

$$k_{t+1} = q\Theta \frac{\Psi\left(\dfrac{\tau B k_t^{\beta}}{\mu_t}\right)}{1+\Psi\left(\dfrac{\tau B k_t^{\beta}}{\mu_t}\right)} k_t^{\beta} \equiv \mathrm{K}(k_t, \mu_t) . \tag{14}$$

Using $Y_t = y_t$ and substituting (2) in (11) yields

$$\mu_{t+1} = \eta\mu_t + pBk_t^{\beta} \equiv \mathrm{M}(k_t, \mu_t), \tag{15}$$

which represents the dynamics of the pollution stock. Thus, the economy's dynamic equilibrium is formally described through

Definition 2. *For* $k_0, \mu_0 > 0$*, the dynamic equilibrium is a sequence of temporary equilibria that satisfy*

(i) $k_{t+1} = \mathrm{K}(k_t, \mu_t)$;

(ii) $\mu_{t+1} = \mathrm{M}(k_t, \mu_t)$.

The economy's long-run equilibrium – that is, its steady-state – is the solution to the planar system of difference equations for the stock of capital per worker and the stock of pollution. Formally, the steady-state equilibrium is a pair $(\hat{k}, \hat{\mu})$ that satisfies $\hat{k} = \mathrm{K}(\hat{k}, \hat{\mu})$ and $\hat{\mu} = \mathrm{M}(\hat{k}, \hat{\mu})$. To obtain it, we use $k_{t+1} = k_t = \hat{k}$ and $\mu_{t+1} = \mu_t = \hat{\mu}$ in equations (14) and (15). Solving (15) for $\hat{\mu}$ yields $\hat{\mu} = \frac{pB}{1-\eta}\hat{k}^{\beta}$. Substituting this solution in (14), and solving for $\hat{k}$, gives

$$\hat{k} = \left\{ q\Theta \frac{\Psi\left[\frac{\tau(1-\eta)}{p}\right]}{1+\Psi\left[\frac{\tau(1-\eta)}{p}\right]} \right\}^{\frac{1}{1-\beta}}. \tag{16}$$

Therefore, the steady-state equilibrium for the pollution stock is

$$\hat{\mu} = \frac{pB}{1-\eta} \left\{ q\Theta \frac{\Psi\left[\frac{\tau(1-\eta)}{p}\right]}{1+\Psi\left[\frac{\tau(1-\eta)}{p}\right]} \right\}^{\frac{\beta}{1-\beta}}. \tag{17}$$

The foregoing analysis provides analytical and explicit solutions for the steady-state values of capital per worker and pollution. Prior to examining the economic implications of varying some structural parameters, we need to determine whether this long-run equilibrium is stable. As it turns out, an additional restriction on the relative share of capital is sufficient to guarantee the stability of the equilibrium. Formally, this is established in

Lemma 1. *Suppose that* $\beta \leq \frac{1}{2}$. *Then the equilibrium pair* $(\hat{k}, \hat{\mu})$, *with* $\hat{k}, \hat{\mu} > 0$, *is locally stable.*

Proof. See the Appendix.

Thus, as long as the share of capital on national income is not high enough, the steady-state equilibrium is non-trivial in the sense that the dynamics starting from any pair of initial values $(k_0 > 0, \mu_0 > 0)$, in the neighbourhood of $(\hat{k}, \hat{\mu})$, will converge to $k_\infty = \hat{k}$ and $\mu_\infty = \hat{\mu}$. Notice that, although it may appear as a limiting scenario, the restriction $\beta \leq \frac{1}{2}$ is supported by numerous empirical estimates who conclude that the relative share of capital income is significantly below 50% (e.g., Poterba, 1998; Gollin, 2002).

The equilibrium can be illustrated by means of the phase diagram in Figure 1. The PS locus is derived from points that satisfy $\mu_t = \mathrm{M}(k_t, \mu_t)$. Clearly, from (15) we get $\mu_t = \frac{pB}{1-\eta} k_t^\beta \equiv \Xi(k_t)$ such that $\Xi' > 0$, $\Xi(0) = 0$ and $\Xi(\infty) \to \infty$. The CS locus is derived from points that satisfy $k_t = \mathrm{K}(k_t, \mu_t)$. Using $k_{t+1} = k_t$ in (14) and rearranging yields $\frac{k_t^{1-\beta}}{\Psi\left(\frac{\tau B k_t^\beta}{\mu_t}\right)(q\Theta - k_t^{1-\beta})} = 1$ or, alternatively, $\Phi(k_t, \mu_t) = 1$. First, we can check that $\Phi_{\mu_t} = \frac{k_t^{1-\beta}}{(q\Theta - k_t^{1-\beta})} \frac{\Psi'}{[\Psi(\cdot)]^2} \frac{\tau B k_t^\beta}{\mu_t^2} > 0$. The next step is to analyse the derivative, $\Phi_{k_t} = \frac{(1-\beta) k_t^{-\beta}}{\Psi(\cdot)(q\Theta - k_t^{1-\beta})} + \frac{k_t^{1-\beta}(1-\beta) k_t^{-\beta}}{\Psi(\cdot)(q\Theta - k_t^{1-\beta})^2} - \frac{k_t^{1-\beta}\Psi'}{[\Psi(\cdot)]^2 (q\Theta - k_t^{1-\beta})} \frac{\beta \tau B k_t^{\beta-1}}{\mu_t}$ or alternatively (after factorisation) $\Phi_{k_t} = \frac{k_t^{-\beta}}{\Psi(\cdot)(q\Theta - k_t^{1-\beta})} \left[1 - \beta + \frac{(1-\beta) k_t^{1-\beta}}{(q\Theta - k_t^{1-\beta})} - \beta \frac{\Psi'}{\Psi(\cdot)} \frac{\tau B k_t^\beta}{\mu_t} \right]$. By assumption, we know that $\frac{\Psi(x_t)}{x_t} > \Psi'(x_t)$ therefore $\frac{1}{x_t} > \frac{\Psi'(x_t)}{\Psi(x_t)}$. If we replace $\frac{1}{x_t}$ for $\frac{\Psi'(x_t)}{\Psi(x_t)}$ in the third term of the expression inside brackets, and then add the first term of the same expression, we get $1-\beta-\frac{\beta}{x_t}\frac{\tau B k_t^{\beta}}{\mu_t}$.[8] After substituting (2), (6) and (8), this expression becomes $1-2\beta$ which is non-negative given that $\beta \leq \frac{1}{2}$ holds by assumption. However, if this expression is non-negative when using $\frac{1}{x_t}$ then it is certainly positive when using $\frac{\Psi'(x_t)}{\Psi(x_t)} < \frac{1}{x_t}$. Consequently, $\Phi_{k_t} > 0$ and equation (15) defines a function $\mu_t \equiv Z(k_t)$ such that $Z' = -\frac{\Phi_{k_t}}{\Phi_{\mu_t}} < 0$. In addition, $\mu_t = 0$ implies $\Psi(\cdot) = \lambda$ and $k_t = \left(q\Theta\frac{\lambda}{1+\lambda}\right)^{\frac{1}{1-\beta}}$ while $\mu_t \to \infty$ implies $\Psi(\cdot) = 0$ and $k_t = 0$. The construction of the diagram is completed by observing that $K_{\mu_t} < 0$ (see the Appendix) and $M_{k_t} > 0$. These imply that above (below) the CS schedule we have $k_{t+1} < k_t$ ($k_{t+1} > k_t$) and on the left (right) of the PS schedule we have $\mu_{t+1} < \mu_t$ ($\mu_{t+1} > \mu_t$).

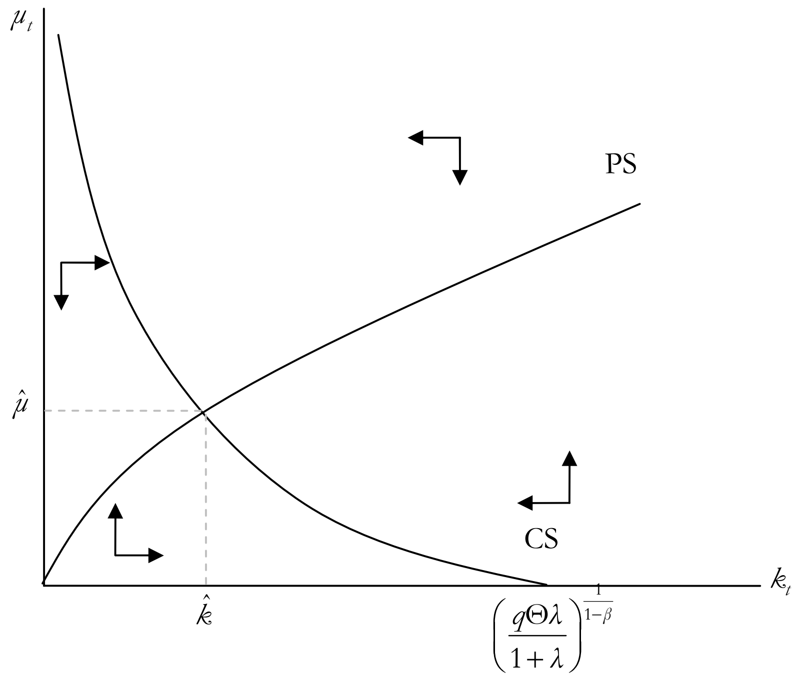

Figure 1. The phase diagram

[8] The second term $\frac{(1-\beta)k_t^{1-\beta}}{(q\Theta-k_t^{1-\beta})}$ is obviously positive.

3.1 Some Comparative Statics

This part of the paper is devoted to the analysis of the equilibrium effects resulting from variations in the model's (policy unrelated) structural parameters. These effects are summarised in

Proposition 1. *An economy with increased emissions and reduced natural absorption capacity will have lower income and higher pollution. More productive technologies are associated with higher income and higher pollution.*

Proof. From (16) we can derive

$$\frac{\partial \hat{k}}{\partial p} = \frac{1}{1-\beta}\left[\frac{q(1-\beta)B(1-\tau)\Psi(\cdot)}{1+\Psi(\cdot)}\right]^{\frac{1}{1-\beta}-1} \frac{q(1-\beta)B(1-\tau)\Psi'(\cdot)}{[1+\Psi(\cdot)]^2}\left[-\frac{\tau(1-\eta)}{p^2}\right] < 0\,,$$

$$\frac{\partial \hat{k}}{\partial \eta} = \frac{1}{1-\beta}\left[\frac{q(1-\beta)B(1-\tau)\Psi(\cdot)}{1+\Psi(\cdot)}\right]^{\frac{1}{1-\beta}-1} \frac{q(1-\beta)B(1-\tau)\Psi'(\cdot)}{[1+\Psi(\cdot)]^2}\left(-\frac{\tau}{p}\right) < 0\,,$$

$$\frac{\partial \hat{k}}{\partial B} = \frac{1}{1-\beta} B^{\frac{1}{1-\beta}-1}\left[\frac{q(1-\beta)(1-\tau)\Psi(\cdot)}{1+\Psi(\cdot)}\right]^{\frac{1}{1-\beta}} > 0\,,$$

and

$$\frac{\partial \hat{k}}{\partial q} = \frac{1}{1-\beta} q^{\frac{1}{1-\beta}-1}\left[\frac{B(1-\beta)(1-\tau)\Psi(\cdot)}{1+\Psi(\cdot)}\right]^{\frac{1}{1-\beta}} > 0\,.$$

From (17) we have

$$\frac{\partial \hat{\mu}}{\partial B} = \frac{p}{1-\eta}\left[\frac{q(1-\beta)B(1-\tau)\Psi(\cdot)}{1+\Psi(\cdot)}\right]^{\frac{\beta}{1-\beta}} + \frac{pB}{1-\eta}\frac{\beta}{1-\beta} B^{\frac{\beta}{1-\beta}-1}\left[\frac{q(1-\beta)(1-\tau)\Psi(\cdot)}{1+\Psi(\cdot)}\right]^{\frac{\beta}{1-\beta}} > 0\,,$$

$$\frac{\partial \hat{\mu}}{\partial q} = \frac{pB}{1-\eta}\frac{\beta}{1-\beta} q^{\frac{\beta}{1-\beta}-1}\left[\frac{B(1-\beta)(1-\tau)\Psi(\cdot)}{1+\Psi(\cdot)}\right]^{\frac{\beta}{1-\beta}} > 0\,,$$

$$\frac{\partial \hat{\mu}}{\partial p} = \frac{B}{1-\eta}\left[\frac{q(1-\beta)B(1-\tau)\Psi(\cdot)}{1+\Psi(\cdot)}\right]^{\frac{\beta}{1-\beta}} + \frac{pB}{1-\eta}\times$$
$$\frac{\beta}{1-\beta}\left[\frac{q(1-\beta)B(1-\tau)\Psi(\cdot)}{1+\Psi(\cdot)}\right]^{\frac{\beta}{1-\beta}-1}\frac{q(1-\beta)B(1-\tau)\Psi'(\cdot)}{[1+\Psi(\cdot)]^2}\left[-\frac{\tau(1-\eta)}{p^2}\right], \tag{18}$$

and

$$\frac{\partial \hat{\mu}}{\partial \eta} = \frac{Bp}{(1-\eta)^2}\left[\frac{q(1-\beta)B(1-\tau)\Psi(\cdot)}{1+\Psi(\cdot)}\right]^{\frac{\beta}{1-\beta}} + \frac{pB}{1-\eta}\times$$
$$\frac{\beta}{1-\beta}\left[q\frac{(1-\beta)B(1-\tau)\Psi(\cdot)}{1+\Psi(\cdot)}\right]^{\frac{\beta}{1-\beta}-1}\frac{q(1-\beta)B(1-\tau)\Psi'(\cdot)}{[1+\Psi(\cdot)]^2}\left(-\frac{\tau}{p}\right). \tag{19}$$

After some manipulation, equations (18) and (19) can be written as

$$\frac{\partial \hat{\mu}}{\partial p} = \frac{B}{1-\eta}\left[\frac{q(1-\beta)B(1-\tau)\Psi(\cdot)}{1+\Psi(\cdot)}\right]^{\frac{\beta}{1-\beta}}\left[1-\frac{\beta}{1-\beta}\frac{\tau(1-\eta)}{p}\frac{\Psi'(\cdot)}{\Psi(\cdot)[1+\Psi(\cdot)]}\right], \tag{20}$$

and

$$\frac{\partial \hat{\mu}}{\partial \eta} = \frac{Bp}{(1-\eta)^2}\left[\frac{q(1-\beta)B(1-\tau)\Psi(\cdot)}{1+\Psi(\cdot)}\right]^{\frac{\beta}{1-\beta}}\left[1-\frac{\beta}{1-\beta}\frac{\tau(1-\eta)}{p}\frac{\Psi'(\cdot)}{\Psi(\cdot)[1+\Psi(\cdot)]}\right], \tag{21}$$

respectively. Now consider the expression $\frac{\beta}{1-\beta}\frac{\tau(1-\eta)}{p}\frac{1}{\hat{x}}$ which, given $\hat{x} = \frac{\tau(1-\eta)}{p}$, equals $\frac{\beta}{1-\beta} \leq 1$ because $\beta \leq 1/2$ holds. However, we know that $\frac{1}{\hat{x}} > \frac{\Psi'(\hat{x})}{\Psi(\hat{x})} > \frac{\Psi'(\hat{x})}{\Psi(\hat{x})[1+\Psi(\hat{x})]}$ holds by assumption. Taking account of equations (20) and (21), we conclude that $\frac{\partial \hat{\mu}}{\partial p} > 0$ and $\frac{\partial \hat{\mu}}{\partial \eta} > 0$. ■

An economy that employs more polluting manufacturing techniques (i.e., higher p) and/or possesses a limited absorption capacity (i.e., higher η) will experience a deterioration of environmental quality. This worsens the health profile of the population, leads to lower life expectancy and acts as an incentive to reduce saving. The process of capital accumulation is impeded and causes a decline in production and, therefore, income. The latter effect imposes some reduction in overall pollutant emissions which is not strong enough, however, to counteract the increase in pollution resulting from the higher rate of emissions per unit manufactured goods. Eventually, the economy will settle down to a new long-run equilibrium with lower income and higher pollution.

An improvement in the productivity of the manufacturing process (i.e., higher B) will promote aggregate savings due to the rise in wages, while an improvement in the efficiency of the financial sector (i.e., higher q) will improve the process of capital formation for given amounts of saving . Both result in greater accumulation of capital which stimulates economic activity. The rise in aggregate production has two conflicting effects on the prospects of longevity. On the one hand, it leads to an increase in the tax revenues which are used to fund the provision of health services. On the other hand, the stimulated activity implies that more pollutants are emitted in the environment. As it turns out, these two conflicting effects will cancel each other out and, eventually, the economy will settle to a new long-run equilibrium with higher income and more pollution.

4 Threshold Effects, Permanent Structural Change and Income-Pollution Dynamics

The previous analysis has demonstrated that the reduction in the emission indicator (i.e., the parameter p) could be crucial in generating an equilibrium that combines higher income and improved environmental conditions. An important aspect that facilitates this result is this frameworks' explicit account of the direct impact of environmental quality on economic activity (through life expectancy). A natural direction of analysis is to consider the possibility that such outcomes may lie at the core of empirical observations which show that, while economic growth is associated with increased environmental degradation at relatively low levels of development, it is also associated with reductions in various measures of pollution at relatively higher levels of development. Both these observations have been commonly associated with the presence of an environmental Kuznets curve (EKC hereafter) – that is, an inverse U-shaped relationship between pollution and income. The purpose of this Section is to provide an alternative explanation for its emergence.

Rather than assuming that the indicator of the technology's dirtiness is a constant parameter, let us consider the case where it is an endogenous variable determined according to $p_t = \varrho(Y_t)$. The idea is that a more developed economy has achieved a level of knowledge, sophistication and expertise, and has the necessary resources to be able to implement a process of pollution abatement that mitigates the damaging effect of economic activity on the environment. In technical terms, this implies that p_t is a negative function of Y_t. This may happen because the government can use part of its revenues towards pollution abatement activities. It could also emerge in a scenario whereby individuals' preferences include a 'warm glow'-type argument inducing them to choose to devote a fraction of their labour earnings for environmentally-friendly activities. For the purposes of this analysis, I will treat the effect of Y_t on p_t as an externality indicating that, as production increases, the participants in the economy become more familiar with certain aspects of the underlying production process – more importantly, with the way through which it demotes environmental quality. This gives them the necessary knowledge on ways to mitigate this damaging impact – knowledge that, subsequently, spreads without cost over the whole economy in the manner of a public good.

To keep matters simple and tractable, I will specify a step function according to which

$$\varrho(Y_t) = \begin{cases} \overline{p} \text{ if } Y_t < \tilde{Y} \\ \\ \underline{p} \text{ if } Y_t \geq \tilde{Y} \end{cases}, \tag{22}$$

where $\overline{p} > \underline{p}$. Again, as production takes place, the participants of the economy acquire knowledge on how to implement a method which minimises the emission generator. In this respect, the threshold could indicate that the economy has accumulated the necessary resources that allow the actual implementation of this technique. Given equation (2) and $Y_t = y_t$, we can write (22) as

$$p_t = \begin{cases} \overline{p} \text{ if } k_t < \tilde{k} \\ \\ \underline{p} \text{ if } k_t \geq \tilde{k} \end{cases}, \tag{23}$$

where $\tilde{k} = (\tilde{Y} / B)^{1/\beta}$. In that case of course, the PS locus will change and take the form of a discontinuous curve, generated by the function $\Xi(k_t) = \frac{p(Bk_t^\beta)B}{1-\eta} k_t^\beta$ for which the expression in (23) indicates that $\lim_{k_t \to \tilde{k}^-} \Xi(k_t) > \lim_{k_t \to \tilde{k}^+} \Xi(k_t)$. Its graph also illustrates the earlier claim that, as long as environmental improvements are bounded, the EKC cannot be explained just by the impact of economic activity on the environment: in this case, the effect of k_t on μ_t resembles an 'N'-shaped rather than an 'inverse-U' shaped curve. Another important implication from this analysis is the possibility of multiple, non-trivial steady-state equilibria, as demonstrated from

Lemma 2. *Suppose that* $\lim_{k_t \to \tilde{k}^-} \Xi(k_t) > Z(\tilde{k}) > \lim_{k_t \to \tilde{k}^+} \Xi(k_t)$ *. Then, there exist two pairs of locally stable steady-state equilibria,* $(\hat{k}^1, \hat{\mu}^1)$ *and* $(\hat{k}^2, \hat{\mu}^2)$*, such that* $\hat{k}^2 > \tilde{k} > \hat{k}^1$ *and* $\hat{\mu}^2 < \hat{\mu}^1$.

Proof. Let us begin with the values for capital intensity that satisfy $k_t < \tilde{k}$. Given $\lim_{k_t \to \tilde{k}^-} \Xi(k_t) > Z(\tilde{k})$, $0 = \Xi(0) < Z(0) \to \infty$, $\Xi'(\cdot) > 0$ and $Z'(\cdot) < 0$, a steady-state equilibrium $(\hat{k}^1, \hat{\mu}^1)$ with $\hat{k}^1 < \tilde{k}$ exists. Analogously, for values of capital intensity that satisfy $k_t \geq \tilde{k}$ we have $\lim_{k_t \to \tilde{k}^+} \Xi(k_t) < Z(\tilde{k})$, $\Xi\left[\left(\frac{q\Theta\lambda}{1+\lambda}\right)^{1/(1-\beta)}\right] > 0 > Z\left[\left(\frac{q\Theta\lambda}{1+\lambda}\right)^{1/(1-\beta)}\right] = 0$, $\Xi'(\cdot) > 0$ and $Z'(\cdot) < 0$. Therefore, a steady-state equilibrium $(\hat{k}^2, \hat{\mu}^2)$ with $\hat{k}^2 > \tilde{k}$ exists. Of course, $Z'(\cdot) < 0$ and $\hat{k}^2 > \hat{k}^1$ imply that $Z(\hat{k}^2) < Z(\hat{k}^1) \Rightarrow \hat{\mu}^2 < \hat{\mu}^1$. Analytically, these pair of equilibria are given by

$$\hat{k}^1 = \left\{ q\Theta \frac{\Psi\left[\frac{\tau(1-\eta)}{\overline{p}}\right]}{1+\Psi\left[\frac{\tau(1-\eta)}{\overline{p}}\right]} \right\}^{\frac{1}{1-\beta}}, \quad \hat{\mu}^1 = \frac{\overline{p}B}{1-\eta} \left\{ q\Theta \frac{\Psi\left[\frac{\tau(1-\eta)}{\overline{p}}\right]}{1+\Psi\left[\frac{\tau(1-\eta)}{\overline{p}}\right]} \right\}^{\frac{\beta}{1-\beta}},$$

and

$$\hat{k}^2 = \left\{ q\Theta \frac{\Psi\left[\dfrac{\tau(1-\eta)}{\underline{p}}\right]}{1+\Psi\left[\dfrac{\tau(1-\eta)}{\underline{p}}\right]} \right\}^{\frac{1}{1-\beta}}, \quad \hat{\mu}^2 = \frac{\underline{p}B}{1-\eta}\left\{ q\Theta \frac{\Psi\left[\dfrac{\tau(1-\eta)}{\underline{p}}\right]}{1+\Psi\left[\dfrac{\tau(1-\eta)}{\underline{p}}\right]} \right\}^{\frac{\beta}{1-\beta}}.$$

By appealing to Proposition 1, we can readily verify that $\hat{k}^2 > \hat{k}^1$ and $\hat{\mu}^2 < \hat{\mu}^1$, while the local stability of these equilibria can be inferred from Lemma 1. ■

The situation described above is illustrated in Figure 2. Given that all other parameters determining the CS schedule are unchanged, the increase in equilibrium income necessitates an improvement in survival prospects brought forward by improvements in environmental quality. The drop in the emission rate is indeed sufficient enough to guarantee that pollutant emissions are lower, even though production is higher for $\hat{k}^2 > \hat{k}^1$. The emergence of multiple equilibria illustrates the important point that it is possible for a more developed country to enjoy better environmental conditions. Of course, the implementation of a technique which is able to reduce the amount of pollutants per unit of produced output is necessary for the existence of this scenario. However, this is not by itself sufficient. Another requirement to ensure this outcome derives from the positive repercussions of reduced pollution for economic activity – exemplified by the negative slope of the CS schedule which illustrates the importance of environmental quality in reducing the risk of early mortality and, thus, promoting capital accumulation. This argument becomes transparent once we consider the case whereby pollution does not impinge on life expectancy: in this case, the CS schedule becomes a line vertical to the horizontal axis at the steady-state level of capital intensity. No matter how sharp the drop in pollutant emissions may be, multiple equilibria cannot exist.

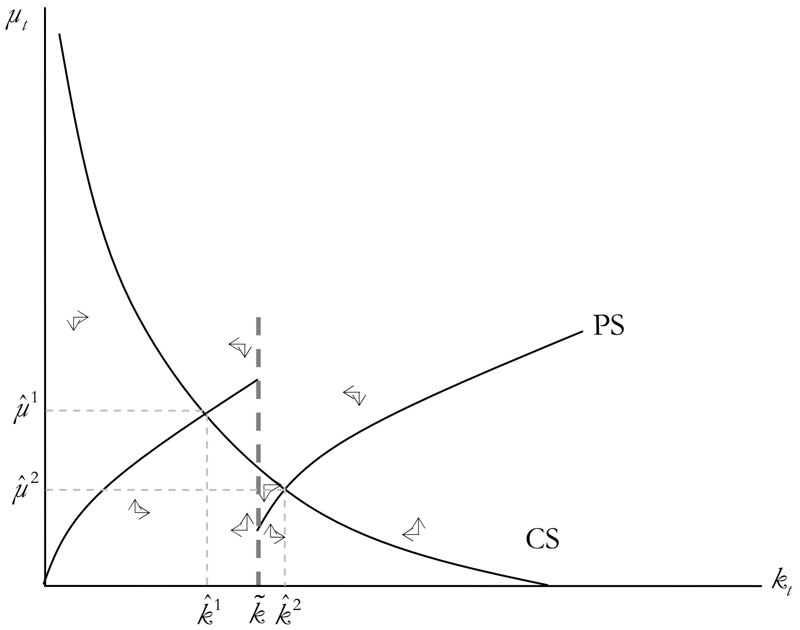

Figure 2. Threshold effects

The situation illustrated in Figure 2 depicts a scenario which is conceptually similar to the type of threshold effects studied by Azariadis and Drazen (1990). Effectively, it implies that only economies with a sufficient endowment of capital will be able to achieve the relatively high income state; otherwise, they will eventually converge to the lower level of equilibrium income. Of course, it may be inappropriate and historically inaccurate to attribute the income differences between industrialised economies and less-developed countries simply to the fact that the former happened to be endowed with more resources when they initiated their process of economic development. A more plausible argument would attribute their better economic performance to certain events and actions (like political, institutional and other structural reforms) that stimulated economic activity in a manner that enhanced their future prospects and allowed them to escape low income traps. In terms of the present framework, one may thing of such events as illustrated by a rightward shift in the CS locus – sufficient enough to guarantee that $\lim_{k_t \to \tilde{k}^-} Z(k_t) < Z(\tilde{k})$. This could allow economies with not such a high initial endowment to surpass the threshold given by $\tilde{k}$.

How may these arguments relate to the joint dynamics of economic development and environmental degradation? My point is that the presence of development thresholds, combined with the idea that the interactions between environmental quality and economic activity are by-directional, could provide an alternative explanation for the observation that led researchers to argue in favour of an EKC when it comes to the linkages between the rate of change in output and changes in environmental conditions. A useful result in understanding these issues is provided with

Proposition 2. *Consider an economy at the equilibrium point* $(\hat{k}^1, \hat{\mu}^1)$ *with* $\hat{k}^1 < \tilde{k}$. *Now suppose that* q *increases so that* $Z(k_t)$ *shifts to* $Z^*(k_t)$ *which is sufficient to guarantee* $\lim_{k_t \to \tilde{k}^-} \Xi(k_t) < Z^*(\tilde{k})$. *As long as* $\Xi(\varkappa) < Z^*(\varkappa)$, *where* $\varkappa = Z^{*(-1)}(\hat{\mu}^1)$, *then the economy will experience a dynamic transition and eventually converge to a new equilibrium point* $(\hat{k}^3, \hat{\mu}^3)$, *such that* $\hat{k}^3 > \tilde{k} > \hat{k}^1$ *and* $\hat{\mu}^3 < \hat{\mu}^1$.

Proof. Since $Z'(\cdot) < 0$ and $\Xi'(\cdot) > 0$, after the shift we have $\lim_{k_t \to \tilde{k}^-} \Xi(k_t) < Z^*(\tilde{k})$ which certainly implies that $\Xi(\hat{k}^1) < Z(\hat{k}^1)$ for $\hat{k}^1 < \tilde{k}$. The new equilibrium for capital intensity must certainly be located above the threshold $\tilde{k}$. Whether the new equilibrium for pollution is below or above the original one depends on whether the point for which $\hat{\mu}^1 = Z^*(\varkappa) \Rightarrow \varkappa = Z^{*(-1)}(\hat{\mu}^1)$ satisfies $\Xi(\varkappa) < Z^*(\varkappa)$ *or* $\Xi(\varkappa) > Z^*(\varkappa)$ respectively. If the former condition holds, then pollution will decline. Thus, $(\hat{k}^3, \hat{\mu}^3)$ satisfies $\hat{k}^3 > \tilde{k} > \hat{k}^1$ and $\hat{\mu}^3 < \hat{\mu}^1$. ■

To understand the transitional dynamics from the original to the new equilibrium, let us examine the graph of Figure 3. Following the permanent improvement in the rate at which resources are transformed into capital (i.e., q), the original equilibrium point $(\hat{k}^1, \hat{\mu}^1)$ becomes, effectively, the initial point of a new dynamic adjustment. Due to the increase in the rate of capital accumulation, income will grow and (for a given emission rate) pollution will increase. Nevertheless, as the economy evolves and reaches $\tilde{k}$, there is a new structural break resulting from the implementation of the 'cleaner' manufacturing method which will lead the economy to a new transition path towards the steady-state equilibrium. As long as the condition described in Proposition 2 holds, pollution will have to decline towards this new steady-state equilibrium. As this happens, the improvement in survival prospects supports an increase in the social marginal product of capital which stimulates the rate of capital accumulation and causes output to grow even further as it converges to its new equilibrium. The latter is an important point in this particular interpretation of the EKC – as this is illustrated by the shape of the transitional dynamics from the original to the new equilibrium. To clarify this, suppose that the drop in the emission rate at $\tilde{k}$ is not possible, meaning that the PS locus is continuously monotonic. In such a case (and for the same increase in q), capital and pollution will increase towards a new equilibrium for which capital intensity will be below $\hat{k}^3$ and pollution above $\hat{\mu}^3$. What this implies is that, after the drop in the emission rate at $\tilde{k}$, it is the reduction in pollution that, to a large extent, causes output to grow above the level justified by the structural change which was originally induced by the permanent rise in q.

Thus, what looks like an EKC does not necessarily imply that the process of economic growth/development is, by itself, sufficient enough to explain the changes in various aspects of environmental quality. To a certain extent, improvements in environmental quality are partially responsible for changes in the processes of economic growth and development as well. The scenario can be summarised in

Corollary 1. *In the transition from* $(\hat{k}^1, \hat{\mu}^1)$ *to* $(\hat{k}^3, \hat{\mu}^3)$, *capital intensity and pollution will initially grow, until the threshold* $\tilde{k}$ *is surpassed. Then pollution will decline and capital intensity will increase further towards the new equilibrium. Thus, for* $k_t < \tilde{k}$, *as income grows it causes pollution to increase while, after the reform which carries the economy above* $\tilde{k}$, *as pollution declines the economy grows even further.*

In other words, the EKC may be the result of the by-directional effects that shape the joint transitional dynamics of pollution and capital accumulation.

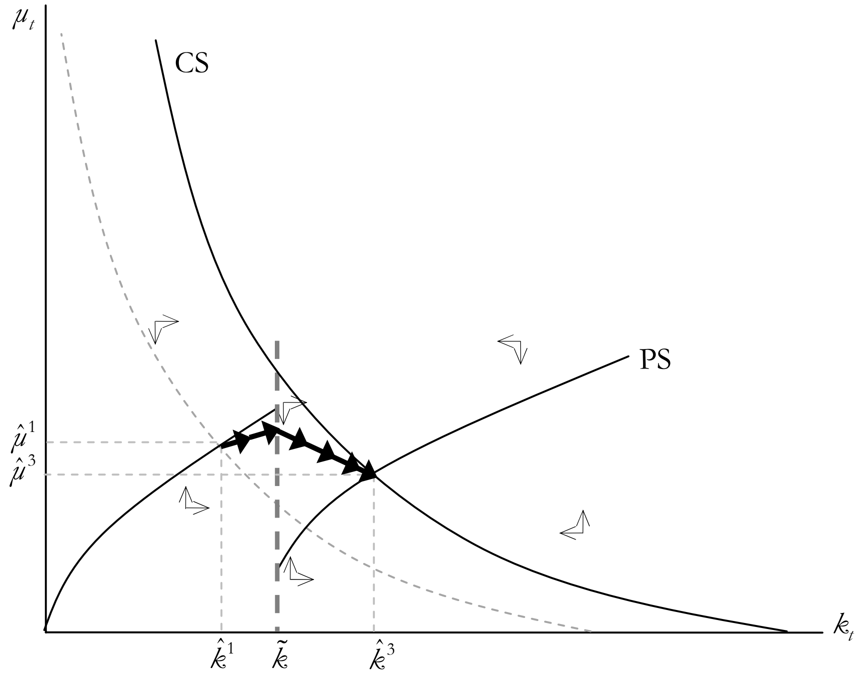

Figure 3. Permanent structural change and transitional dynamics

5 Health Spending and Economic Development

As we have already seen, the government imposes a proportional tax on production and uses the proceeds in order to finance the provision of public health services. In this Section, I consider the effects of this policy on the development prospects of the economy. For this reason, I will return my attention to the situation in which the economy generates the unique, interior equilibrium given in (16) and (17).

While considering the impact of taxation on capital accumulation and income we have to bear in mind that there are two conflicting effects at work. On the one hand, an increase in taxation crowds out private investment by reducing the amount of funds available for saving. On the other hand, it allows the government to provide more essential health services which promote capital accumulation because they reduce the risk of untimely death. The implication from these combined effects is formally described in

Lemma 3. *There is a unique tax rate* $\tau^* \in (0,1)$ *that maximises equilibrium income. As a result it satisfies* $\partial \hat{k} / \partial \tau^* > 0$ *for* $\tau < \tau^*$ *and* $\partial \hat{k} / \partial \tau^* < 0$ *for* $\tau > \tau^*$.

Proof. Equation (16) reveals that the two limiting cases of $\tau = 0$ and $\tau = 1$ lead to $\hat{k} = 0$. Using this equation, we can also derive

$$\frac{\partial \hat{k}}{\partial \tau} = \frac{[q(1-\beta)B]^{\frac{1}{1-\beta}}}{1-\beta}\left[\frac{(1-\tau)\Psi(\cdot)}{1+\Psi(\cdot)}\right]^{\frac{1}{1-\beta}-1}\left\{-\frac{\Psi(\cdot)}{1+\Psi(\cdot)}+(1-\tau)\frac{\Psi'(\cdot)}{[1+\Psi(\cdot)]^2}\frac{(1-\eta)}{p}\right\}.$$

If we factorise with $1/[1+\Psi(\cdot)]$ we can clearly see that the sign of this expression depends on the sign of

$$-\Psi(\cdot)+(1-\tau)\frac{\Psi'(\cdot)}{1+\Psi(\cdot)}\frac{(1-\eta)}{p} \equiv \Omega. \tag{22}$$

We can check that, for $\tau = 0$, we have $\Psi(\cdot) = 0$, $\Psi'(\cdot) = \lambda$ and, therefore, equation (22) is positive. Similarly, when $\tau = 1$ we have $\Psi(\cdot) > 0$ therefore equation (22) is negative. Furthermore, we can use (22) to check the derivative

$$\frac{\partial \Omega}{\partial \tau} = -\frac{\Psi'(\cdot)(1-\eta)}{p} - \frac{\Psi'(\cdot)}{1+\Psi(\cdot)}\frac{(1-\eta)}{p} + \frac{(1-\tau)\Psi''(\cdot)}{1+\Psi(\cdot)}\left[\frac{(1-\eta)}{p}\right]^2 - \frac{(1-\tau)[\Psi'(\cdot)]^2}{[1+\Psi(\cdot)]^2}\left[\frac{(1-\eta)}{p}\right]^2,$$

which is clearly negative given that $\Psi' > 0$ and $\Psi'' < 0$. We conclude that there exists some τ^* such that $\partial \hat{k} / \partial \tau^* = 0$, $\partial \hat{k} / \partial \tau^* > 0$ for $\tau < \tau^*$ and $\partial \hat{k} / \partial \tau^* < 0$ for $\tau > \tau^*$. ■

When $\tau < \tau^*$ ($\tau > \tau^*$), the benefit (in terms of higher life expectancy) of a marginal increase in the tax rate is higher (lower) that the corresponding cost, which takes the form of the reduction in funds available for saving. Naturally, there exists a tax rate which balances these two effects and can lead to the maximum equilibrium level for capital intensity and income. Although τ^* is evidently related to the parameters that determine environmental quality, it is not possible to determine (with certainty) how the income-maximising tax rate responds to variations in these parameters, when using the general specification for the survival probability. However, it is possible to determine these effects after specifying a functional form for $\Psi(\cdot)$. The result is summarised in

Proposition 3. *Suppose* $\Psi(\cdot) = \frac{\lambda x_t}{1+x_t}$. *Then* $\tau^* = \mathrm{T}(p, \eta)$ *such that* $\mathrm{T}_p, \mathrm{T}_\eta > 0$.

Proof. Using (2), (6), (8), (16) and (17), we can establish that, in the steady-state, we have $\hat{x}=\frac{\tau(1-\eta)}{p}$. Combining with $\Psi(\cdot)=\frac{\lambda\hat{x}}{1+\hat{x}}$ and substituting in (16) yields

$$\hat{k}=\left[q(1-\beta)B(1-\tau)\frac{\lambda\tau(1-\eta)}{p+(1+\lambda)\tau(1-\eta)}\right]^{\frac{1}{1-\beta}}. \tag{23}$$

Therefore,

$$\frac{\partial\hat{k}}{\partial\tau}=\frac{[q(1-\beta)B]^{\frac{1}{1-\beta}}}{1-\beta}\left[\frac{(1-\tau)\lambda\tau(1-\eta)}{p+(1+\lambda)\tau(1-\eta)}\right]^{\frac{1}{1-\beta}-1}\left\{-\frac{\lambda\tau(1-\eta)}{p+(1+\lambda)\tau(1-\eta)}+\frac{p(1-\tau)\lambda(1-\eta)}{[p+(1+\lambda)\tau(1-\eta)]^2}\right\}.$$

Since $\tau=0$ and $\tau=1$ cannot be maximising choices for $\hat{k}$, τ^* is derived by setting $\{\cdot\}=0$. Therefore,

$$\frac{\lambda\tau^*(1-\eta)}{p+(1+\lambda)\tau^*(1-\eta)}=\frac{p(1-\tau^*)\lambda(1-\eta)}{[p+(1+\lambda)\tau^*(1-\eta)]^2}\Rightarrow$$

$$\tau^*=\frac{p(1-\tau^*)}{p+(1+\lambda)\tau^*(1-\eta)}\Rightarrow$$

$$(\tau^*)^2(1+\lambda)(1-\eta)+2p\tau^*-p=0. \tag{24}$$

The expression in (24) is a quadratic equation with only one positive solution

$$\tau^*=\sqrt{\gamma+\gamma^2}-\gamma,$$

where $\gamma=\frac{p}{(1+\lambda)(1-\eta)}$. Clearly, $0<\sqrt{\gamma+\gamma^2}-\gamma<1\ \ \forall\gamma>0$ and

$$\frac{\partial\tau^*}{\partial\gamma}=\frac{1}{2}(\gamma+\gamma^2)^{-\frac{1}{2}}(1+2\gamma)-1.$$

It is

$$\frac{1}{2}(\gamma+\gamma^2)^{-\frac{1}{2}}(1+2\gamma)>1\Rightarrow$$

$$(\gamma+\gamma^2)^{-\frac{1}{2}}(1+2\gamma)>2\Rightarrow$$

$$(1+2\gamma)>2(\gamma+\gamma^2)^{\frac{1}{2}}\Rightarrow$$

$$(1+2\gamma)^2>4(\gamma+\gamma^2)\Rightarrow$$

$$1+4\gamma+4\gamma^2>4\gamma+4\gamma^2,$$

which is true, meaning that $\frac{\partial \tau^*}{\partial \gamma} > 0$. Furthermore, it is $\frac{\partial \gamma}{\partial p}, \frac{\partial \gamma}{\partial \eta} > 0$. Hence, we conclude that $\frac{\partial \tau^*}{\partial p} > 0$ and $\frac{\partial \tau^*}{\partial \eta} > 0$. ■

The above analysis allows us to make an inference in the form of

Corollary 2. *Under a reasonable specification for the survival probability, and as long as tax proceeds are used productively, a change in structural characteristics indicating greater environmental degradation imply that an increase in taxation may support the economy's long-run development prospects.*

A rise in the parameter values that indicate greater environmental degradation, will reduce the marginal losses and will have an ambiguous effect in the marginal gains resulting from a higher tax rate. This is because the parameters p and η lead to a decline of life expectancy (reducing the cost, in terms of foregone expected income, due to higher taxation) and impede the overall health profile which, given diminishing returns for $\Psi(\cdot)$, implies an increase in the benefit from a marginal rise in longevity. Despite the fact that p and η are also responsible for a reduction in the direct benefit of providing health services, when $\Psi(\cdot) = \frac{\lambda x_t}{1 + x_t}$ the former effects dominate and the most conducive tax rate, in terms of economic development, is positively related with the parameters p and η.

6 Conclusions

In this paper, I have constructed a model in which the dynamics of pollution and capital accumulation interact and are, therefore, jointly determined. While capital accumulation is responsible for the built-up of more pollutants, the latter reduce capital formation due to their detrimental effect on life expectancy and, therefore, saving behaviour. I have shown a scenario whereby these joint dynamics provide a different explanation for the observed co-movements of per capita GDP and pollution – co-movements that have been associated with the presence of an environmental Kuznets curve. I have also used this framework to examine how variations in environment-related parameters determine the effect of taxation in economic development, when tax revenues contribute to the provision of public health care.

The mechanisms and the intuition behind all the results indicate the importance of having a consistent account of the impact of environmental quality on economic activity. Naturally, when considering various factors that may link pollution to economic growth, the health status emerges as a prominent candidate – it is affected by the quality of the environment and it certainly affects economic behaviour, decisions and, ultimately, outcomes. In addition to variations in life expectancy, the detrimental effect of environmental degradation on the health characteristics of the population may be channelled to economic growth as a result of variations in labour productivity or variations in the ability of agents to undertake a task with direct effects on the economic environment – Gradus and Smulders (1993) have, for example, considered the possibility that a polluted environment can influence the ability of agents to accumulate human capital through deliberate learning activities. Undoubtedly, the explicit modelling of all these ideas will enrich our understanding of many important issues pertaining to the growth-environment nexus and, thus, represent a fruitful avenue for future research work.

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Appendix

Proof of Lemma 1

The Jacobian matrix associated with the dynamical system of (14) and (15) is

$$\begin{pmatrix} \mathrm{K}_{k_t}(\hat{k},\hat{\mu}) & \mathrm{K}_{\mu_t}(\hat{k},\hat{\mu}) \\ \mathrm{M}_{k_t}(\hat{k},\hat{\mu}) & \mathrm{M}_{\mu_t}(\hat{k},\hat{\mu}) \end{pmatrix}.$$

The trace and the determinant are given by $T = \mathrm{K}_{k_t}(\hat{k},\hat{\mu}) + \mathrm{M}_{\mu_t}(\hat{k},\hat{\mu})$ and $D = \mathrm{K}_{k_t}(\hat{k},\hat{\mu})\mathrm{M}_{\mu_t}(\hat{k},\hat{\mu}) - \mathrm{K}_{\mu_t}(\hat{k},\hat{\mu})\mathrm{M}_{k_t}(\hat{k},\hat{\mu})$ respectively. It is well known (e.g., de la Croix and Michel, 2002) that the stability of the equilibrium is established when the conditions $(1+D-T)(1+D+T) > 0$ and $|D| < 1$ hold simultaneously.

From equation (14), we have

$$\mathrm{K}_{k_t}(\hat{k},\hat{\mu}) = q\Theta\left\{\beta\hat{k}^{\beta-1}\frac{\Psi(\cdot)}{1+\Psi(\cdot)} + \hat{k}^{\beta}\frac{\Psi'(\cdot)}{[1+\Psi(\cdot)]^2}\frac{\beta\tau B\hat{k}^{\beta-1}}{\hat{\mu}}\right\} > 0\,. \tag{A1}$$

Substituting (16) and (17) in (A1) yields

$$\mathrm{K}_{k_t}(\hat{k},\hat{\mu}) = \beta q\Theta\left\{(q\Theta)^{\frac{\beta-1}{1-\beta}}\left[\frac{\Psi(\cdot)}{1+\Psi(\cdot)}\right]^{\frac{\beta-1}{1-\beta}}\frac{\Psi(\cdot)}{1+\Psi(\cdot)} + \frac{\Psi'(\cdot)}{[1+\Psi(\cdot)]^2}\frac{\tau B\hat{k}^{2\beta-1}}{\frac{pB}{1-\eta}\hat{k}^{\beta}}\right\} \Rightarrow$$

$$\mathrm{K}_{k_t}(\hat{k},\hat{\mu}) = \beta q\Theta\left\{\frac{1}{q\Theta} + \frac{\Psi'(\cdot)}{[1+\Psi(\cdot)]^2}\frac{\tau(1-\eta)\hat{k}^{\beta-1}}{p}\right\} \Rightarrow$$

$$\mathrm{K}_{k_t}(\hat{k},\hat{\mu}) = \beta q\Theta\left\{\frac{1}{q\Theta} + \frac{\Psi'(\cdot)}{[1+\Psi(\cdot)]^2}\frac{\tau(1-\eta)}{p}(q\Theta)^{\frac{\beta-1}{1-\beta}}\left[\frac{\Psi(\cdot)}{1+\Psi(\cdot)}\right]^{\frac{\beta-1}{1-\beta}}\right\} \Rightarrow$$

$$\mathrm{K}_{k_t}(\hat{k},\hat{\mu}) = \beta\left\{1 + \frac{\Psi'(\cdot)}{\Psi(\cdot)[1+\Psi(\cdot)]}\frac{\tau(1-\eta)}{p}\right\}. \tag{A2}$$

Let us consider the expression

$$\beta\left[1 + \frac{1}{\hat{x}}\frac{\tau(1-\eta)}{p}\right]. \tag{A3}$$

In the steady-state we have $\hat{x} = \dfrac{\hat{h}}{\hat{\mu}} = \dfrac{\tau B\hat{k}^{\beta}}{pB\hat{k}^{\beta}/(1-\eta)} = \dfrac{\tau(1-\eta)}{p}$ therefore (A3) becomes 2β. Of course, $2\beta \leq 1$ given that $\beta \leq 1/2$ by assumption. But since $\dfrac{\Psi(\hat{x})}{\hat{x}} > \Psi'(\hat{x})$ also holds by assumption, then $\dfrac{1}{\hat{x}} > \dfrac{\Psi'(\hat{x})}{\Psi(\hat{x})} > \dfrac{\Psi'(\hat{x})}{\Psi(\hat{x})[1+\Psi(\hat{x})]}$. Consequently, if (A3) cannot take a value above unity then, from (A2), it is certainly $0 < \mathrm{K}_{k_t}(\hat{k},\hat{\mu}) < 1$.

Using equation (15) we get $\mathrm{M}_{\mu_t}(\hat{k},\hat{\mu}) = \eta \in (0,1)$ which implies that $T = \eta + \mathrm{K}_{k_t}(\hat{k},\hat{\mu}) > 0$. Furthermore, we can use (14) and (15) to derive

$$\mathrm{M}_{k_t}(\hat{k},\hat{\mu}) = p\beta B\hat{k}^{\beta-1} > 0, \tag{A4}$$

and

$$\mathrm{K}_{\mu_t}(\hat{k},\hat{\mu}) = q\Theta\hat{k}^{\beta}\frac{\Psi'(\cdot)}{[1+\Psi(\cdot)]^2}\left(-\frac{\tau B\hat{k}^{\beta}}{\hat{\mu}^2}\right) < 0. \tag{A5}$$

Thus, (A4) and (A5), combined with previous results, imply that $D = \eta\mathrm{K}_{k_t}(\hat{k},\hat{\mu}) - \mathrm{K}_{\mu_t}(\hat{k},\hat{\mu})\mathrm{M}_{k_t}(\hat{k},\hat{\mu}) > 0$ and $1+D+T > 0$. Additionally, we can derive

$$D - T + 1 = \eta\mathrm{K}_{k_t}(\hat{k},\hat{\mu}) - \mathrm{K}_{\mu_t}(\hat{k},\hat{\mu})\mathrm{M}_{k_t}(\hat{k},\hat{\mu}) - \eta - \mathrm{K}_{k_t}(\hat{k},\hat{\mu}) + 1 \Rightarrow$$

$$D - T + 1 = 1 - \eta - (1-\eta)\mathrm{K}_{k_t}(\hat{k},\hat{\mu}) - \mathrm{K}_{\mu_t}(\hat{k},\hat{\mu})\mathrm{M}_{k_t}(\hat{k},\hat{\mu}) \Rightarrow$$

$$D - T + 1 = (1-\eta)[1 - \mathrm{K}_{k_t}(\hat{k},\hat{\mu})] - \mathrm{K}_{\mu_t}(\hat{k},\hat{\mu})\mathrm{M}_{k_t}(\hat{k},\hat{\mu}).$$

Given (A4), (A5) and $0 < \mathrm{K}_{k_t}(\hat{k},\hat{\mu}) < 1$, we have $D - T + 1 > 0$ which means that $(D+T+1)(D-T+1) > 0$. Consequently, since $D > 0$, we need to show that $D < 1$ in order to establish the stability of the equilibrium.

Substitution of (17) in (A5) yields

$$\mathrm{K}_{\mu_t}(\hat{k},\hat{\mu}) = -q\Theta\frac{\Psi'(\cdot)}{[1+\Psi(\cdot)]^2}\frac{\tau B\hat{k}^{2\beta}}{(pB)^2\hat{k}^{2\beta}/(1-\eta)^2} \Rightarrow$$

$$\mathrm{K}_{\mu_t}(\hat{k},\hat{\mu}) = \frac{-q\Theta\tau(1-\eta)^2}{p^2 B}\frac{\Psi'(\cdot)}{[1+\Psi(\cdot)]^2}. \tag{A6}$$

Using (16) in (A4) yields

$$\mathrm{M}_{k_t}(\hat{k},\hat{\mu}) = p\beta B(q\Theta)^{\frac{\beta-1}{1-\beta}}\left[\frac{\Psi(\cdot)}{1+\Psi(\cdot)}\right]^{\frac{\beta-1}{1-\beta}} \Rightarrow$$

$$\mathrm{M}_{k_t}(\hat{k},\hat{\mu}) = \frac{p\beta B}{q\Theta}\frac{1+\Psi(\cdot)}{\Psi(\cdot)}. \tag{A7}$$

Combining (A6) and (A7), we can derive

$$\mathrm{K}_{\mu_t}(\hat{k},\hat{\mu})\mathrm{M}_{k_t}(\hat{k},\hat{\mu}) = \frac{-q\Theta\tau(1-\eta)^2}{p^2 B}\frac{\Psi'(\cdot)}{[1+\Psi(\cdot)]^2}\frac{p\beta B}{q\Theta}\frac{1+\Psi(\cdot)}{\Psi(\cdot)} \Rightarrow$$

$$\mathrm{K}_{\mu_t}(\hat{k},\hat{\mu})\mathrm{M}_{k_t}(\hat{k},\hat{\mu}) = \frac{-\beta\tau(1-\eta)^2}{p}\frac{\Psi'(\cdot)}{\Psi(\cdot)[1+\Psi(\cdot)]}. \tag{A8}$$

Next, we can combine (A2) and (A8) to derive the determinant

$$D = \eta\mathrm{K}_{k_t}(\hat{k},\hat{\mu}) - \mathrm{K}_{\mu_t}(\hat{k},\hat{\mu})\mathrm{M}_{k_t}(\hat{k},\hat{\mu}) \Rightarrow$$

$$D = \eta\beta + \eta\beta\frac{\Psi'(\cdot)}{\Psi(\cdot)[1+\Psi(\cdot)]}\frac{\tau(1-\eta)}{p} + \frac{\beta\tau(1-\eta)^2}{p}\frac{\Psi'(\cdot)}{\Psi(\cdot)[1+\Psi(\cdot)]} \Rightarrow$$

$$D = \beta\left\{\eta + \frac{\Psi'(\cdot)}{\Psi(\cdot)[1+\Psi(\cdot)]}\frac{\tau(1-\eta)}{p}[\eta + (1-\eta)]\right\} \Rightarrow$$

$$D = \beta\left\{\eta + \frac{\Psi'(\cdot)}{\Psi(\cdot)[1+\Psi(\cdot)]}\frac{\tau(1-\eta)}{p}\right\}. \tag{A9}$$

Now, consider the expression

$$\beta\left[\eta + \frac{1}{\hat{x}}\frac{\tau(1-\eta)}{p}\right]. \tag{A10}$$

In the steady-state we have $\hat{x} = \frac{\tau(1-\eta)}{p}$. Substituting in (A10) yields $\beta(1+\eta) < 1$ because $\beta \leq 1/2$ and $0 < \eta < 1$. However, it is $\frac{1}{\hat{x}} > \frac{\Psi'(\hat{x})}{\Psi(\hat{x})} > \frac{\Psi'(\hat{x})}{\Psi(\hat{x})[1+\Psi(\hat{x})]}$ because $\frac{\Psi(\hat{x})}{\hat{x}} > \Psi'(\hat{x})$ holds by assumption. This implies that, if (A10) is below 1, then, given (A9), we can conclude that $D < 1$ as well. Hence, we have proven that the equilibrium $\hat{k},\hat{\mu} > 0$ is locally stable. ■